Revisão

Produto de vetores

$\hookrightarrow$ escalar $\longrightarrow$

$\hookrightarrow$ vetorial

$\theta = \widehat{uv}$

def $\rightarrow$ Teorema do cosseno

$$\langle u, v \rangle = \begin{cases} \|u\| \cdot \|v\| \cos\theta & u \neq 0 \quad v \neq 0 \\ 0 & \text{se } u \text{ ou } v = 0 \end{cases}$$

$\Longrightarrow \langle u, v \rangle \rightarrow \quad u = u_1 \hat{i} + u_2 \hat{j} + u_3 \hat{k}$

prop. distributiva

$$\boxed{\begin{matrix} \hat{i} \cdot \hat{i} = 1 & i \cdot j = 0 & i \cdot k = 0 \\ & & \hat{k} \cdot \hat{k} = 0 \\ \hat{j} \cdot \hat{k} = 0 & \hat{j} \cdot \hat{j} = 1 & \end{matrix}}$$

$$\underline{\langle u, v \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3}$$

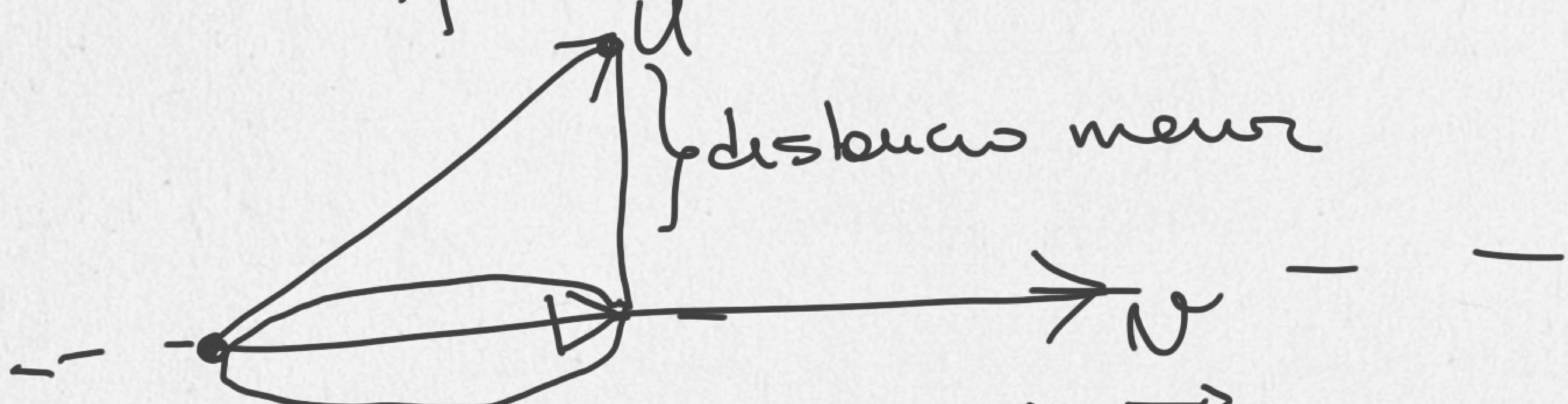
$\langle u, v \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3$

$$\boxed{\frac{\langle u, v \rangle}{\|u\| \, \|v\|} = \cos\theta}$$

Analítica

Achar angulo entre u e v

↳ $\langle u, v \rangle = 0 \iff u \perp v$

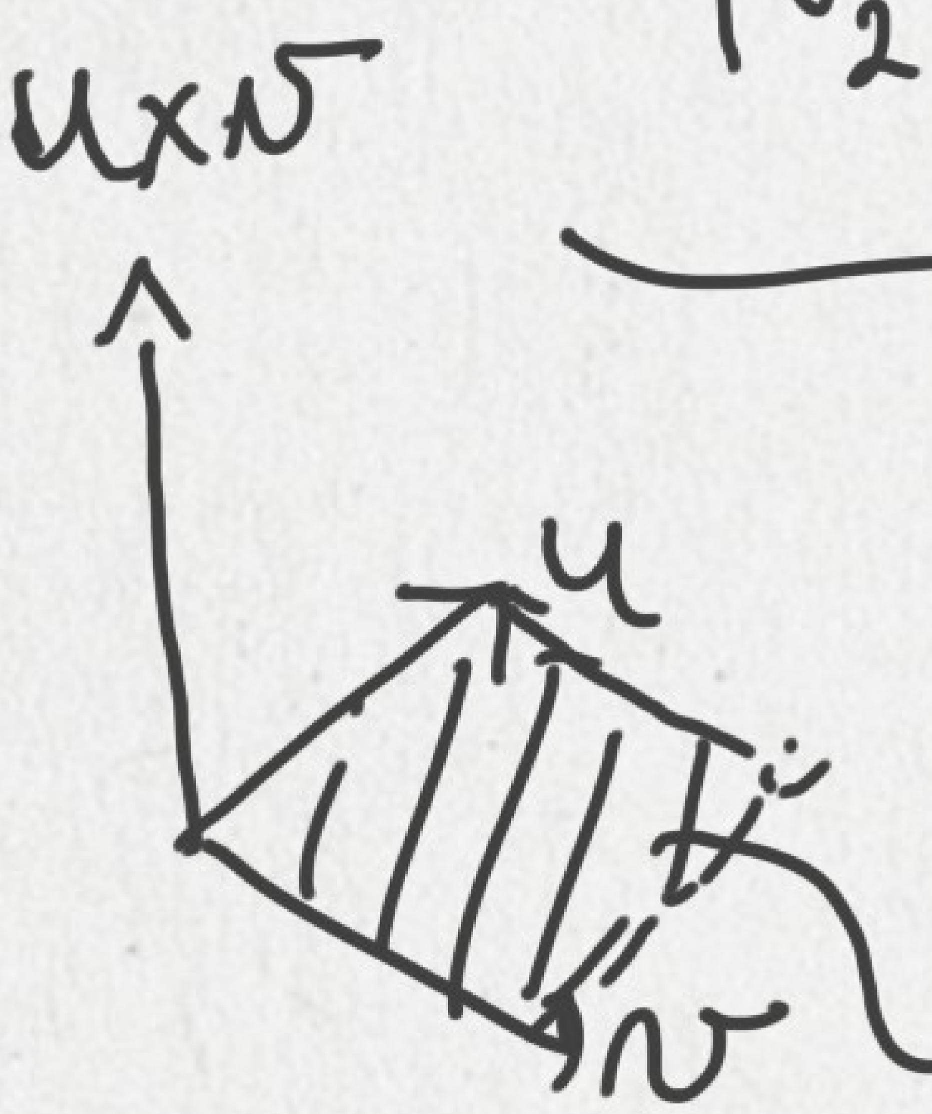


$$\text{Proj}_{v}^{u} = \frac{\langle u, v \rangle}{\langle u, u \rangle} \vec{v}$$

Produto vetorial

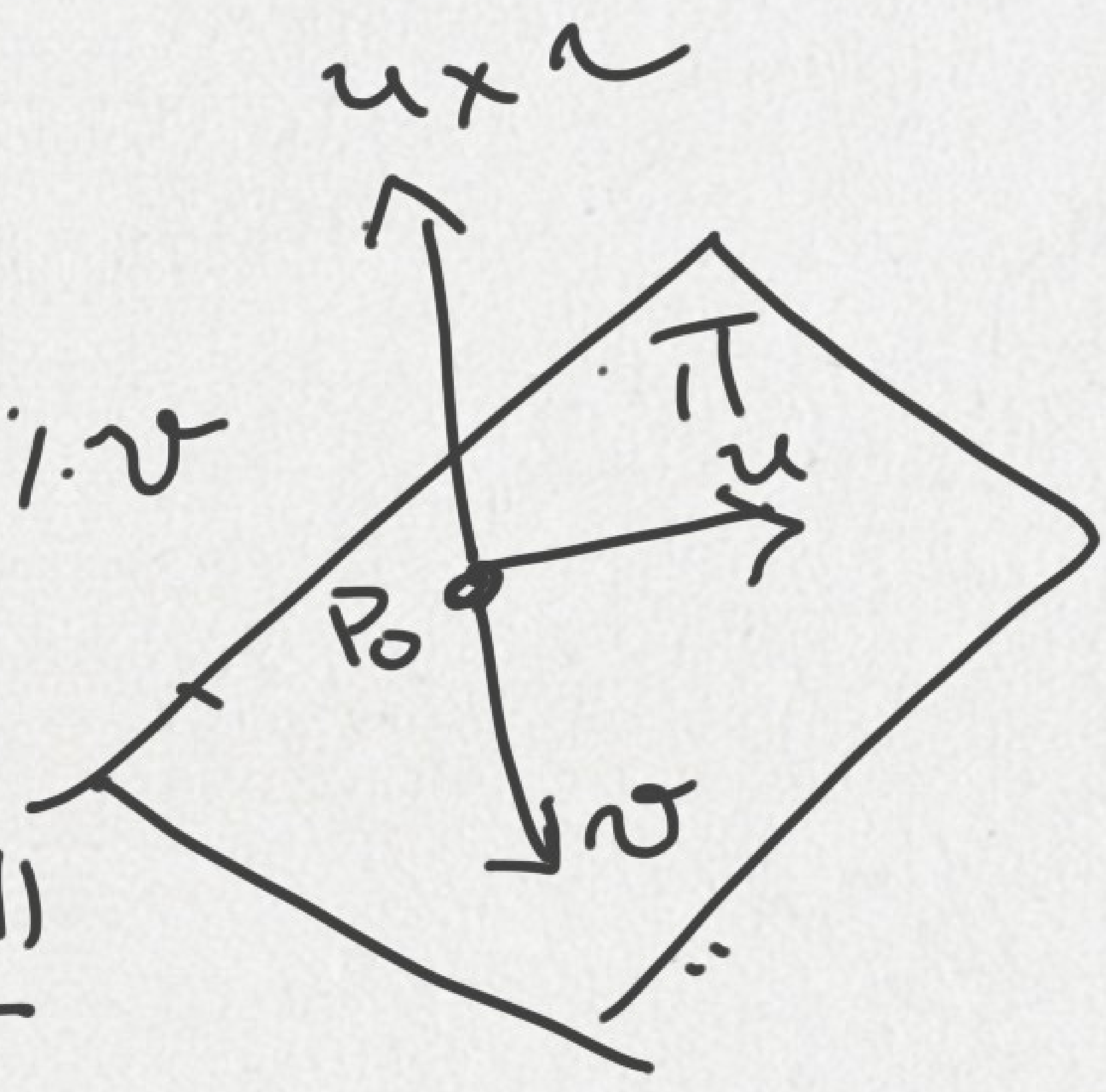
$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} =$$

$$\underbrace{\hat{i}\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - j\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + k\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}}$$



$$\|u \times v\| = \|u\| \cdot \|v\| \operatorname{sen} \widehat{uv}$$

$u \nparallel v$

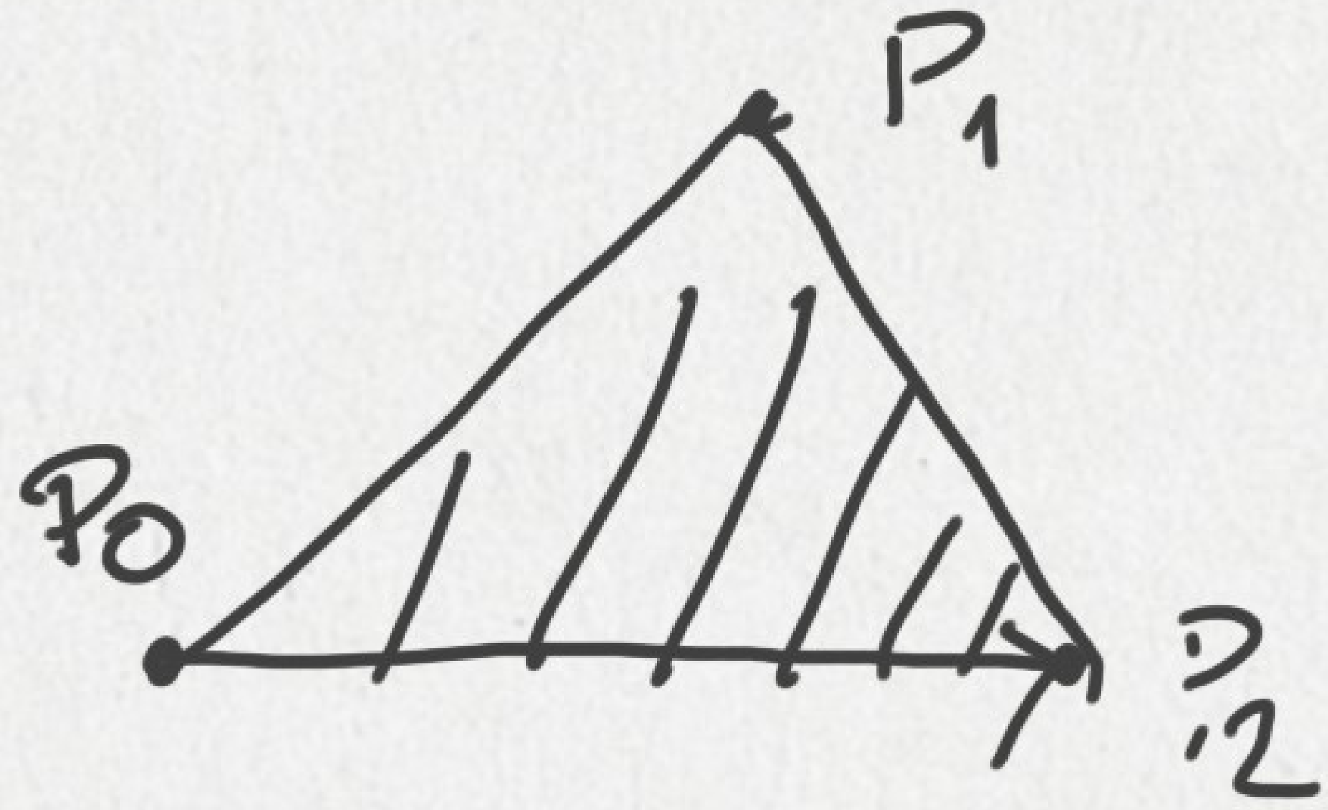


$u \times v \perp u$

$u \times v \perp v$

$\Rightarrow$

$$A = \frac{\|u \times v\|}{2}$$

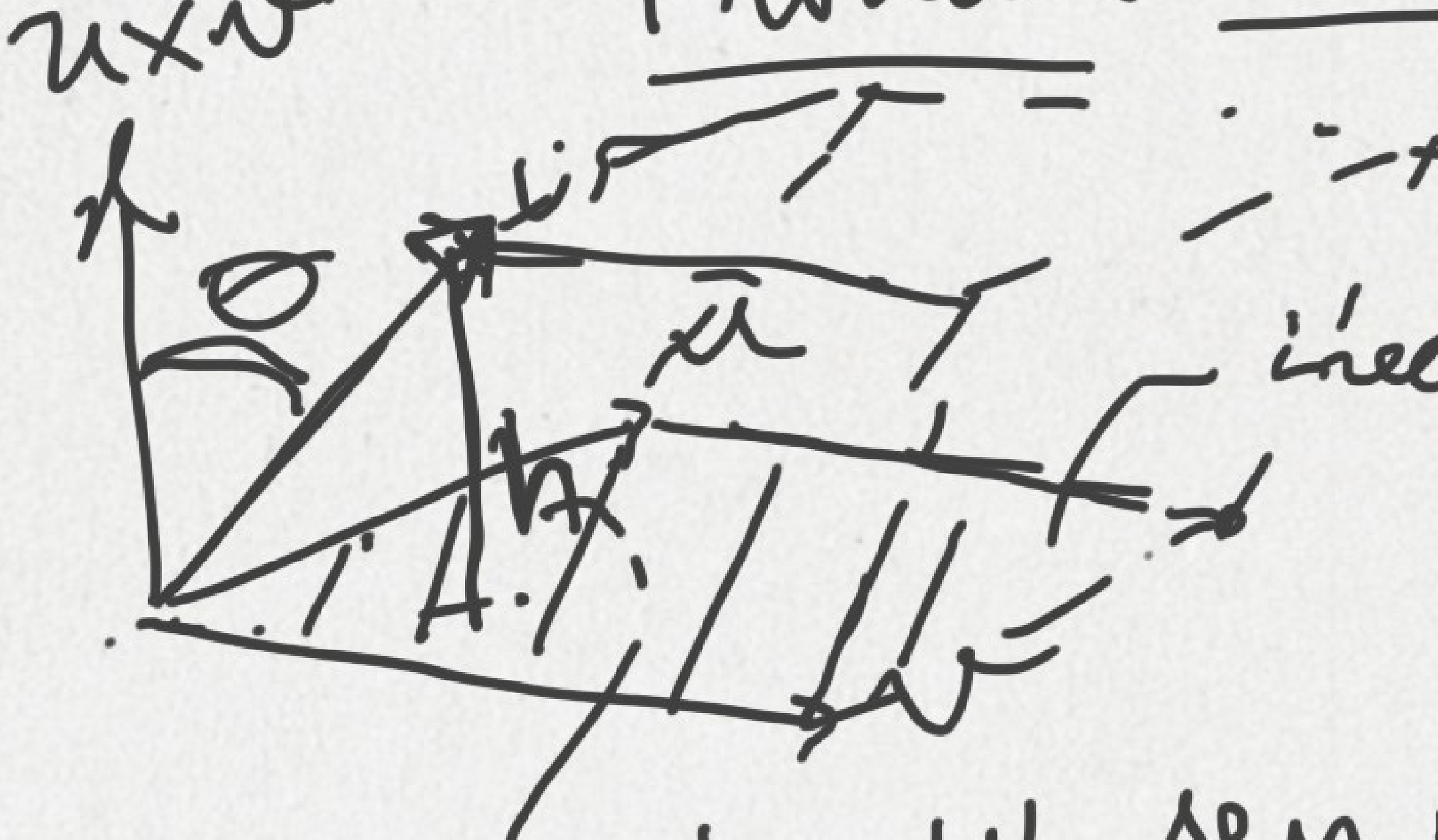


$$u = \overline{P_0P_1}, \quad v = \overline{P_2P_0} \quad \Bigg\} \quad \frac{\|u \times v\|}{2}$$

$$u \parallel v \iff u \times v = 0$$

$$u \parallel v \iff \exists \alpha \text{ t.q. } u = \alpha v$$

Produto misto



$$\widehat{w \;\; u \times v}$$

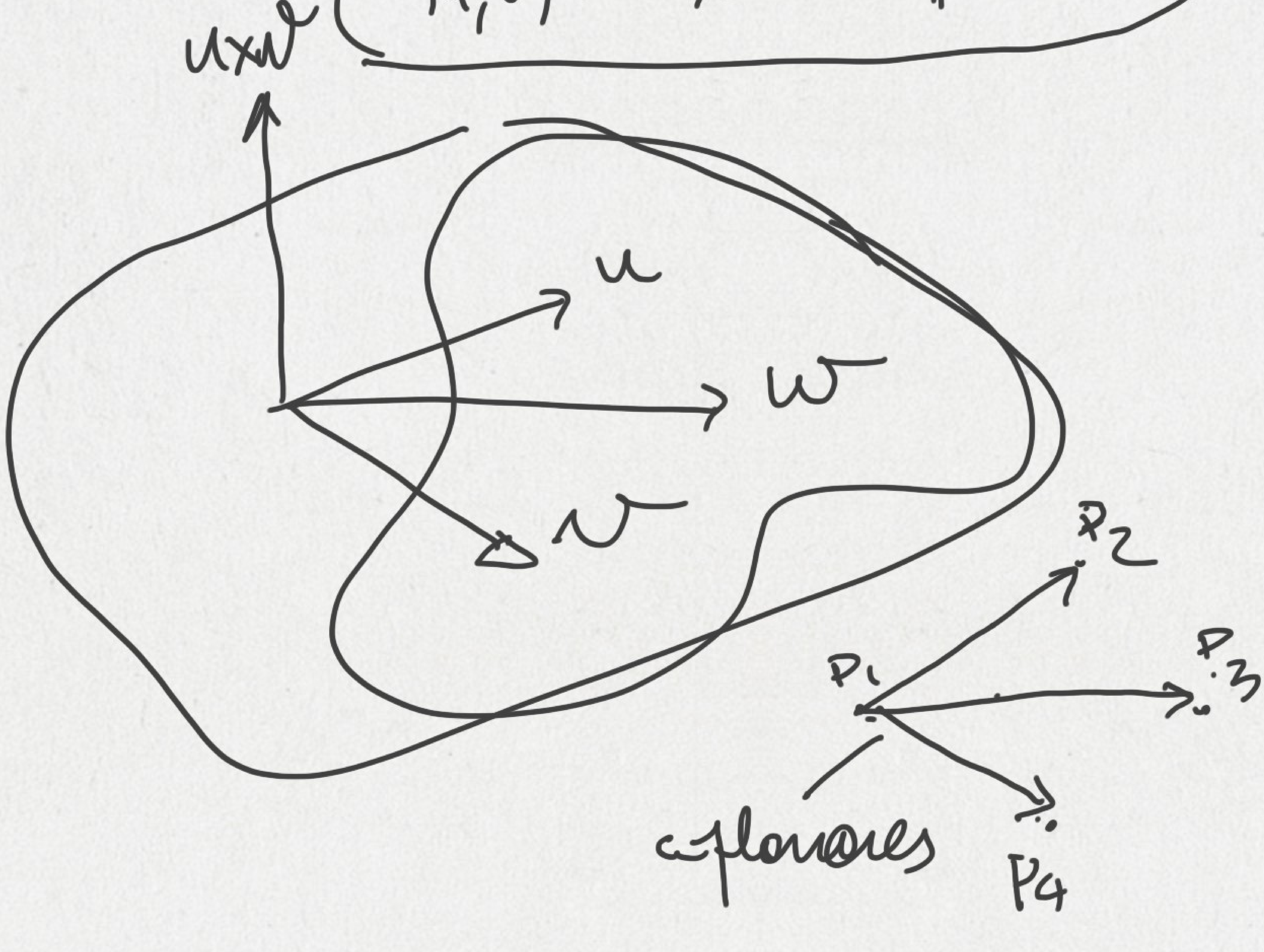
$$h = w \operatorname{sen} \theta$$

$$\underbrace{|\langle (u\times v), w\rangle|}_{\text{escalar}} = \underline{\text{volume}}$$

do paralelepípedo

$\langle (u\times v, w\rangle = 0 \Longrightarrow$

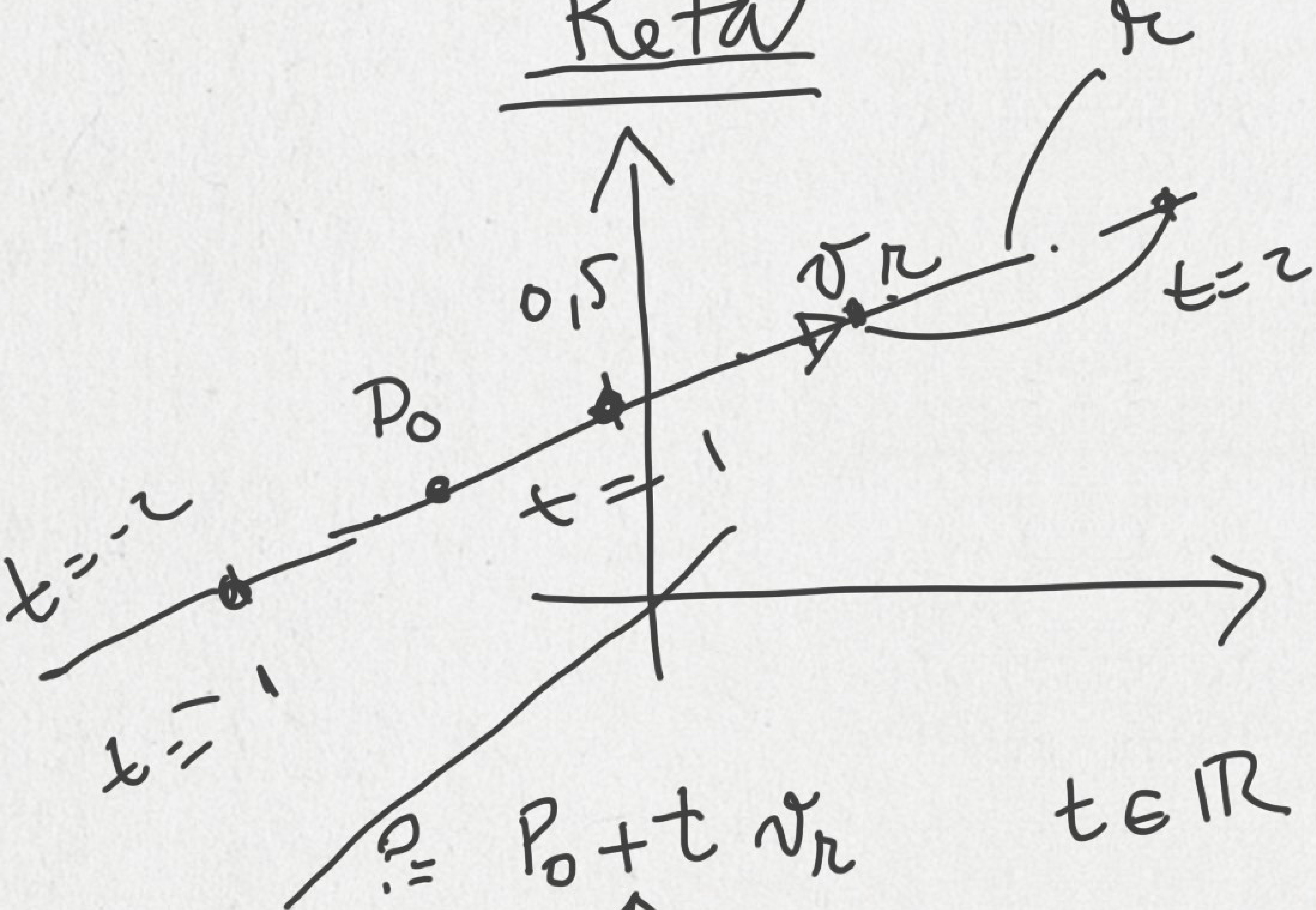
u, v, w são coplanares



$(u\times v)\times w \neq u\times(v\times w)$

exercício

Reta



$P = P_0 + t\,v_r \qquad t\in\mathbb{R}$

$\boxed{(x,y,3)}$

↳ caracterizo todos os pontos da reta

$P_0 \in r$ e v_r vetor diretor

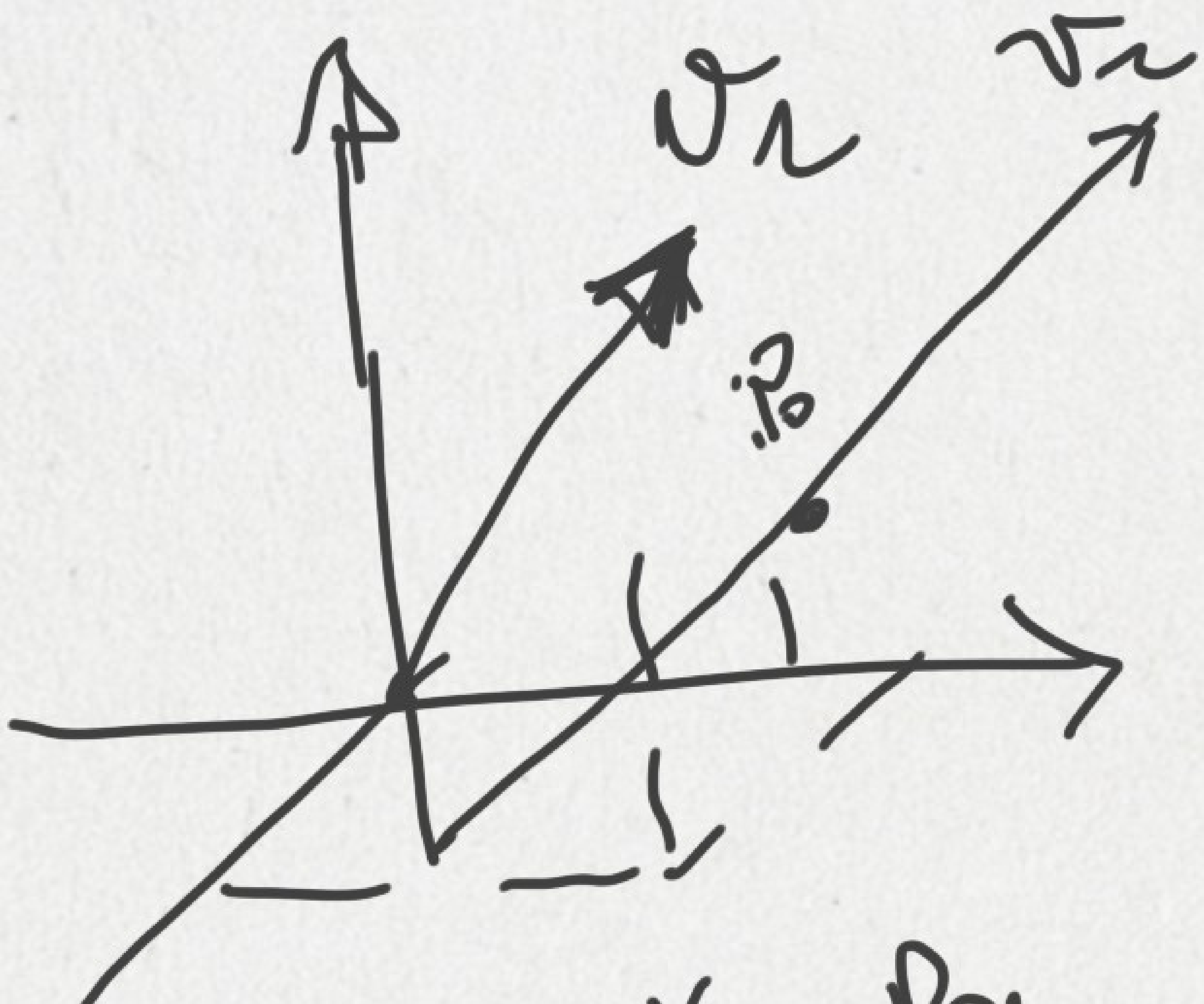
P_0 e v_r vetor diretor

$$(x,y,z) = P_0 + t\,(v_{r1}, v_{r2}, v_{r3})$$

$$\begin{cases} x = P_{0_1} + t\, v_{r1} \\ y = P_{0_2} + t\, v_{r2} \\ z = P_{0_3} + t\, v_{r3} \end{cases}$$

Eq paramétrica da reta

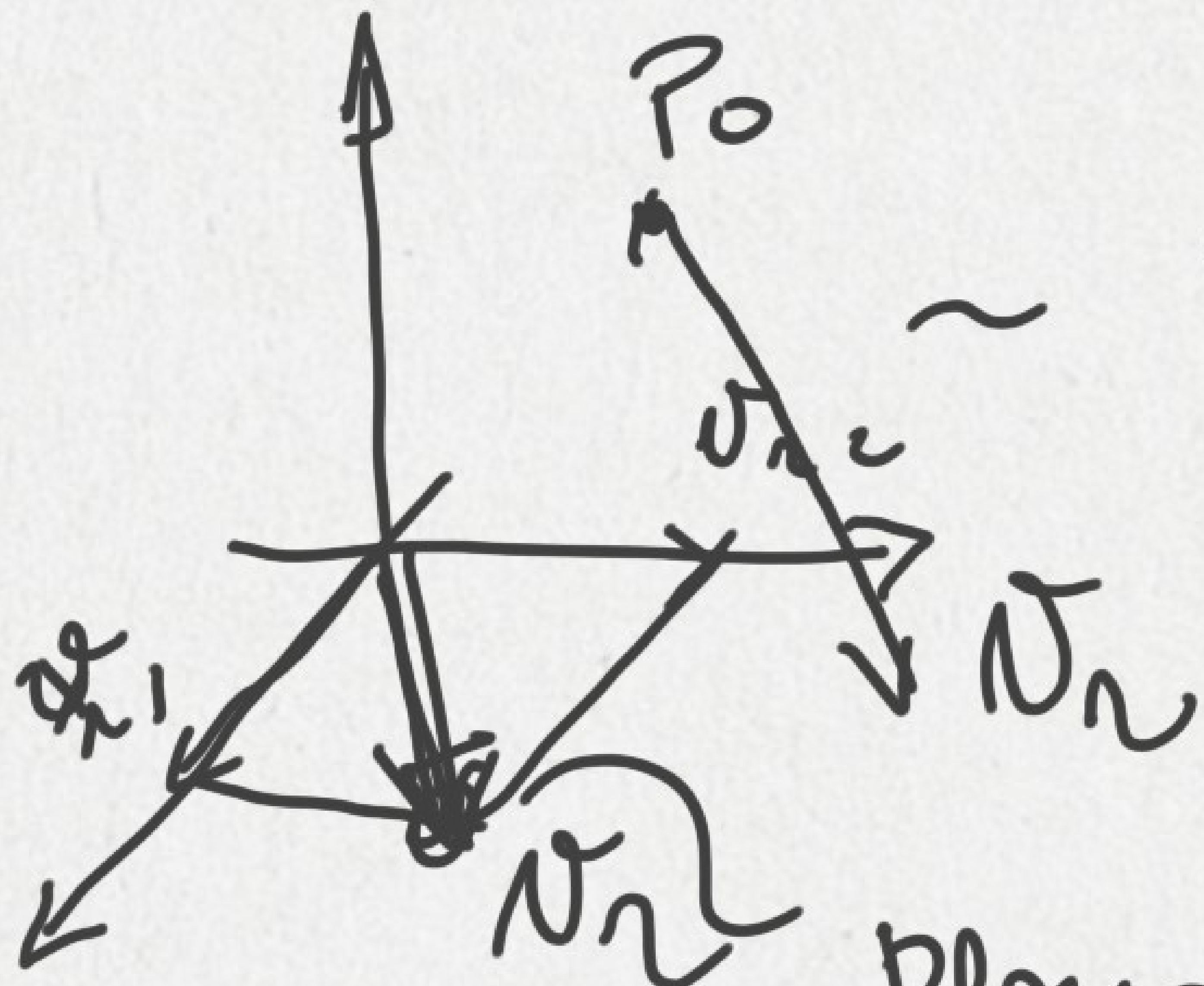
$v_{r1} \neq 0$, $v_{r2} \neq 0$ e $v_{r3} \neq 0$



Equação simétrica

$$t = \frac{x - P_{01}}{v_{r1}} = \frac{y - P_{02}}{v_{r2}} = \frac{z - P_{03}}{v_{r3}}$$

$v_{r1} \neq 0 \quad v_{r2} \neq 0 \quad v_{r3} = 0$



~ Paralelo ao plano(x, y)

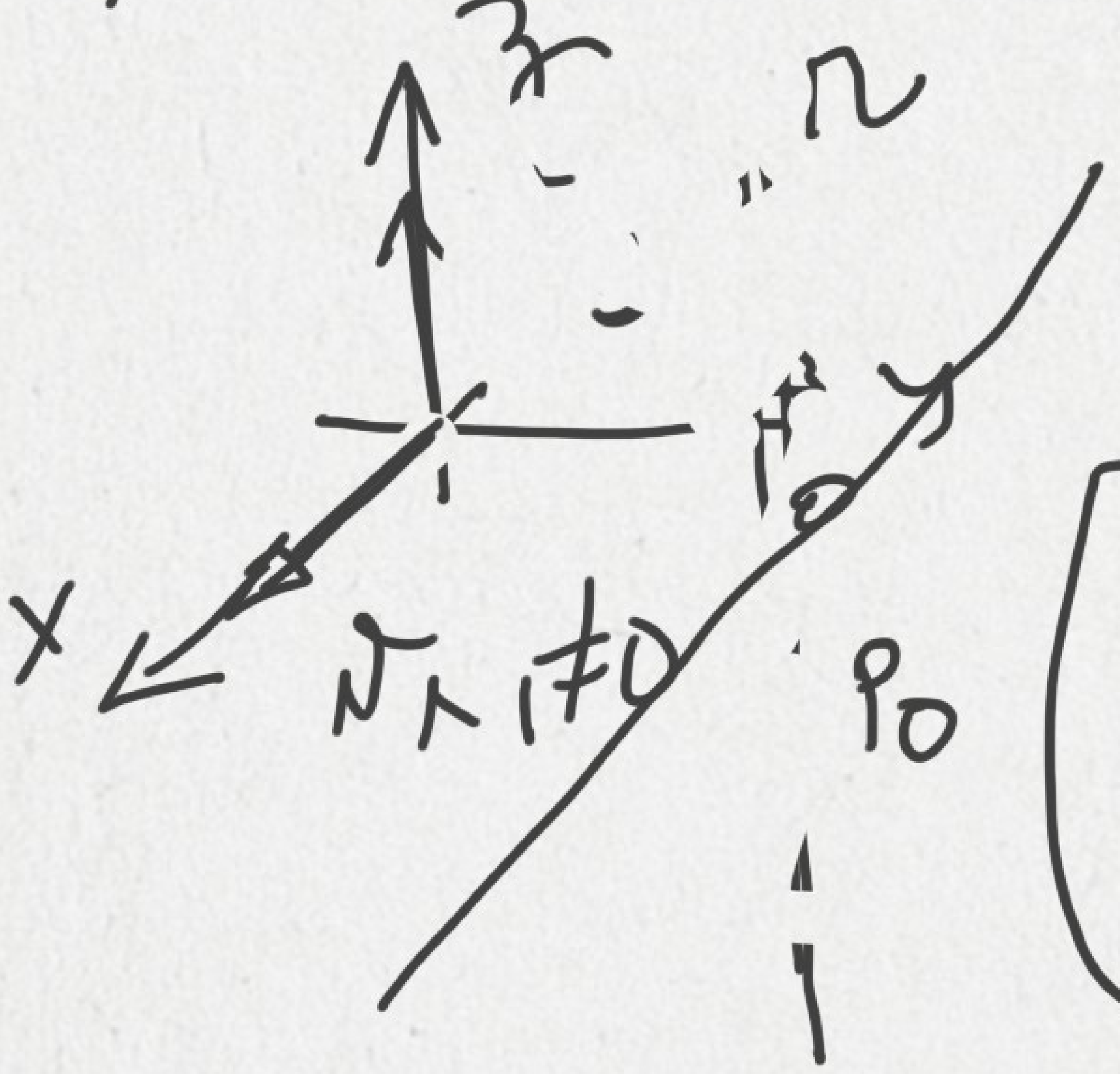
plano (x, y)

$$\begin{cases} x = P_{01} + t\, v_{r1} \\ y = P_{02} + t\, v_{r2} \\ z = P_{03} \end{cases}$$

$\Downarrow$

$$\boxed{\frac{x - P_{01}}{v_{r1}} = \frac{y - P_{02}}{v_{r2}} \qquad z = P_{03}}$$

$v_{r1} \neq 0$, $v_{r2} = 0$, $v_{r3} = 0$



$x = P_{01} + t\, v_{r1}$

$$\boxed{\begin{aligned} y &= P_{02} \\ z &= P_{03} \end{aligned}}$$

$$\boxed{x \in \mathbb{R}, \quad y = P_{02} \quad z = P_{03}}$$

$x = t$

$$\Rightarrow \boxed{2x - 3 = -3y + 2 \qquad z = 4}$$

Escrever a equação Paramétrica e a equação simétrica

$$2x-3 = 2\left(x-\tfrac{3}{2}\right) = \frac{x-3/2}{1/2}$$

$$-3y+2 = -3\left(y-\tfrac{2}{3}\right) = \frac{y-2/3}{-1/3}$$

Simetrica

$$\boxed{\frac{x-3/2}{1/2} = \frac{y-2/3}{-1/3} \qquad z=4}$$

$t = 1-\alpha$

$$\begin{cases} x = \frac{3}{2} + \frac{1}{2}t \\ y = \frac{2}{3} - \frac{1}{3}t \\ z = 4 \end{cases}$$

Parametrica

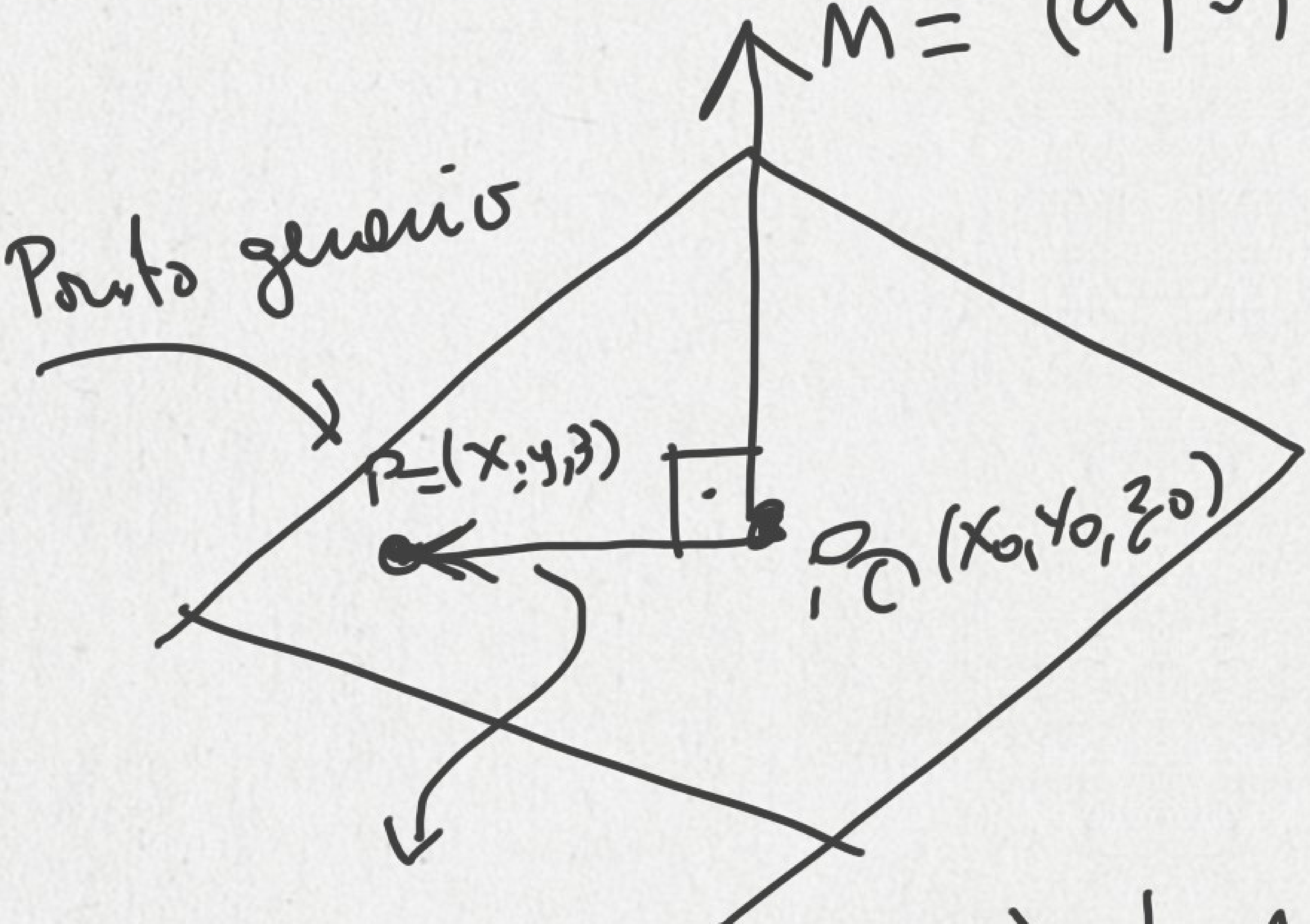
$$\begin{cases} x = \frac{3}{2} + \frac{1}{2} - \frac{1}{2}\alpha \\ y = \frac{2}{3} - \frac{1}{3} + \frac{1}{3}\alpha \\ z = 4 \end{cases}$$

$$\begin{cases} x = 2 - \frac{1}{2}\alpha \\ y = \frac{1}{3} + \frac{1}{3}\alpha \\ z = 4 \end{cases}$$

Planos

vetor normal $\vec{n}$, $P_0 \in \pi$

$n = (a, b, c)$



$$\underbrace{(x - x_0,\ y - y_0,\ z - z_0) \perp n}_{\text{Produto escalar } = 0}$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$\boxed{ax + by + cz + d = 0}$$

Eq cartesiana $\qquad d = -ax_0 - by_0 - cz_0$

$$\boxed{ax + by + cz + d = 0}$$

$$\vec{n} = (a, b, c)$$

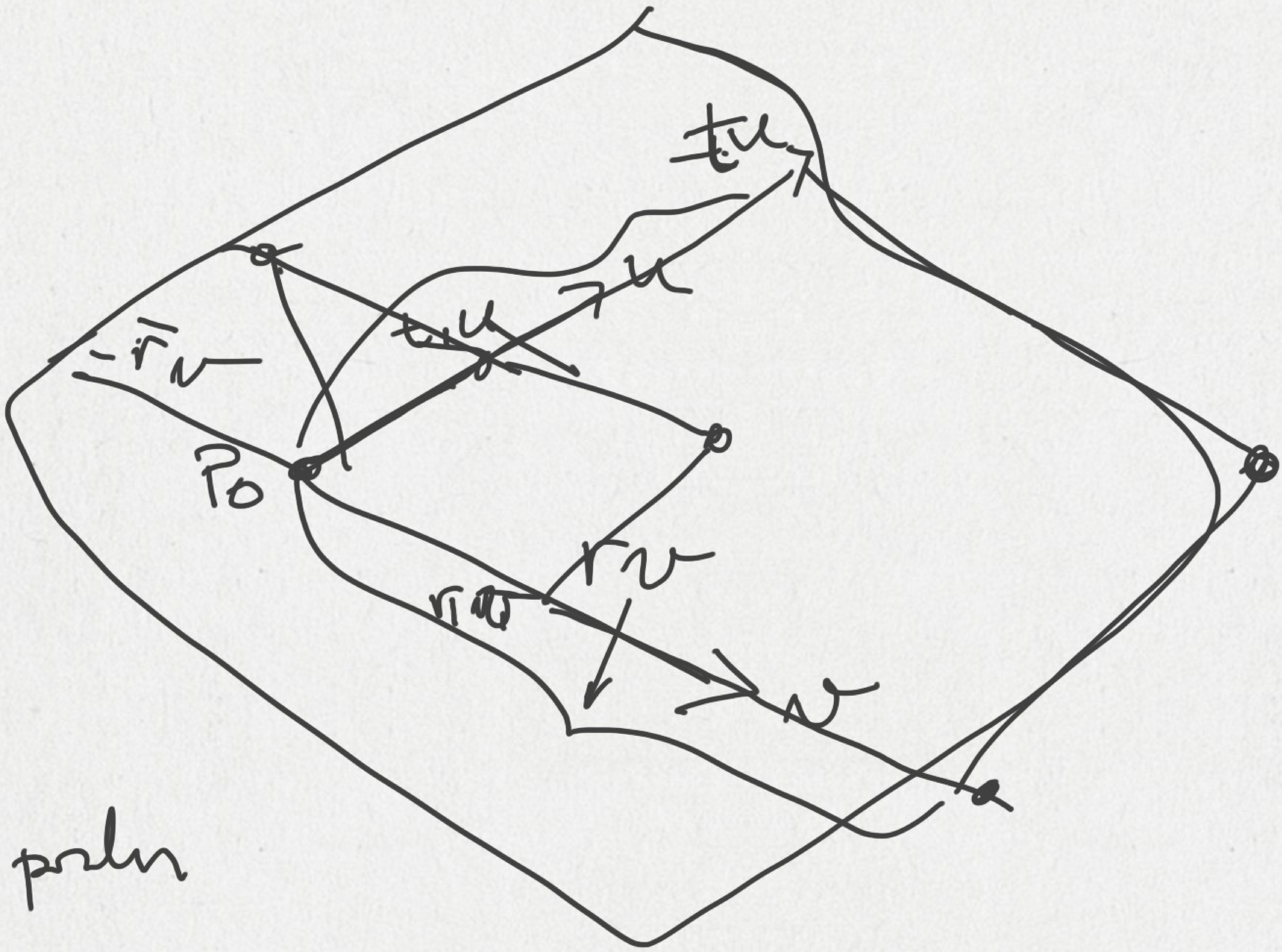
$a \neq 0 \quad y = 0,\ z = 0 \Rightarrow$

$$ax + b(0) + c(0) + d = 0$$

$$x = -\frac{d}{a}$$

$$\left(-\frac{d}{a}, 0, 0\right) \in \text{Plano}$$

Equação Paramétrica



pnln

$$P = P_0 + t\vec{u} + s\vec{v}$$

(x, y, z)

$$\begin{cases} x = x_0 + t u_1 + s v_1 \\ y = y_0 + t u_2 + s v_2 \\ z = z_0 + t u_3 + s v_3 \end{cases}$$

Eq Paramétrica do Plano

dado o plano

$$\begin{cases} x = -2 + t \\ y = t - s \\ z = -3 + t + 2s \end{cases}$$

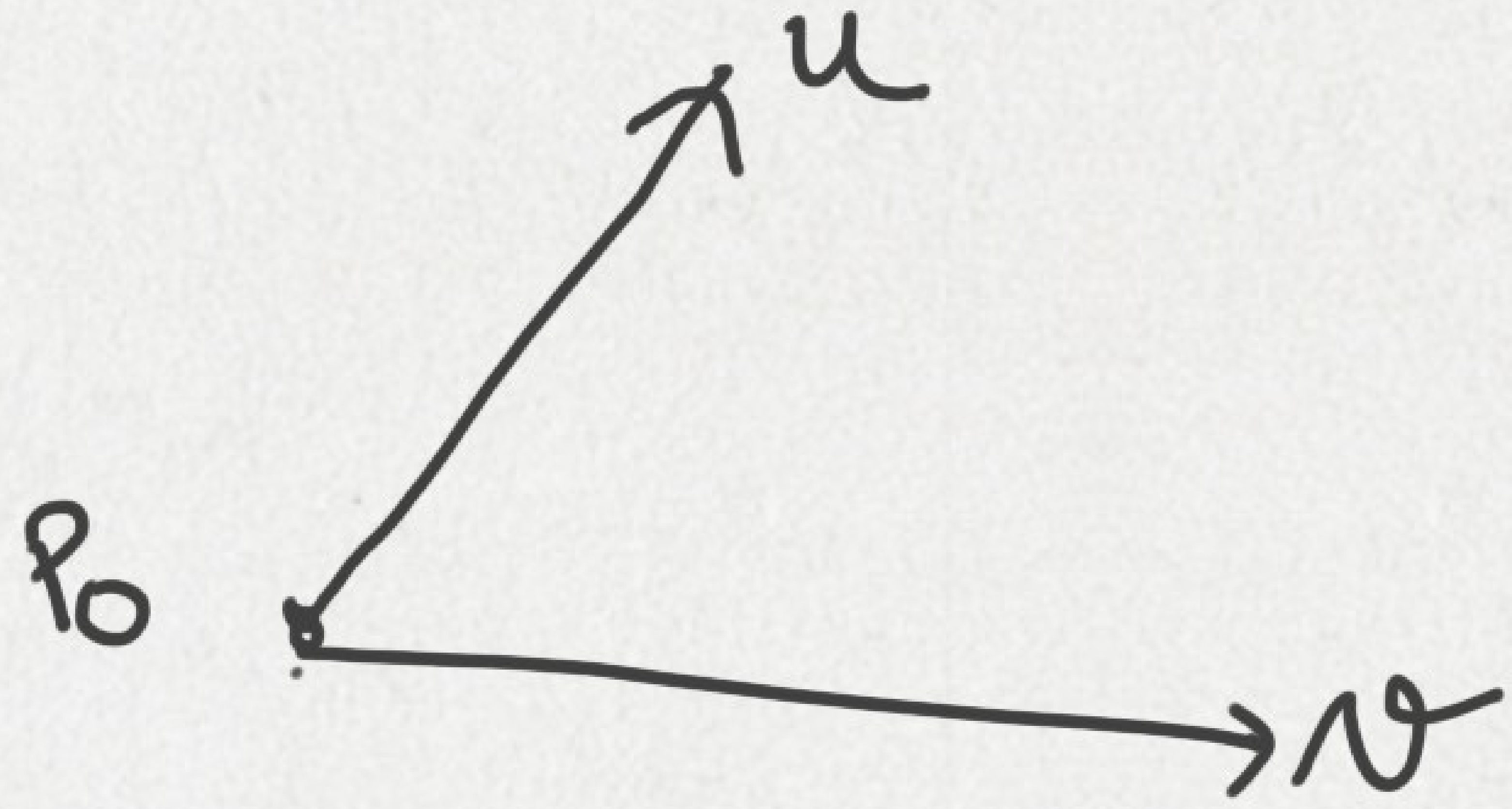
$P_0 + t u + s v$

eq paramétrica.

$\Longrightarrow$ Achar a eq cartesiana

$P_0 = (-2, 0, -3)$ $\quad \vec{u} = (1, 1, 1)$

$\vec{v} = (0, -1, 2)$



$$\vec{M} = u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & -1 & 2 \end{vmatrix} =$$

$$= 3\hat{i} - 2\hat{j} - k \qquad P_0 = (-2, 0, -3)$$

$$\vec{n} = (3, -2, -1)$$

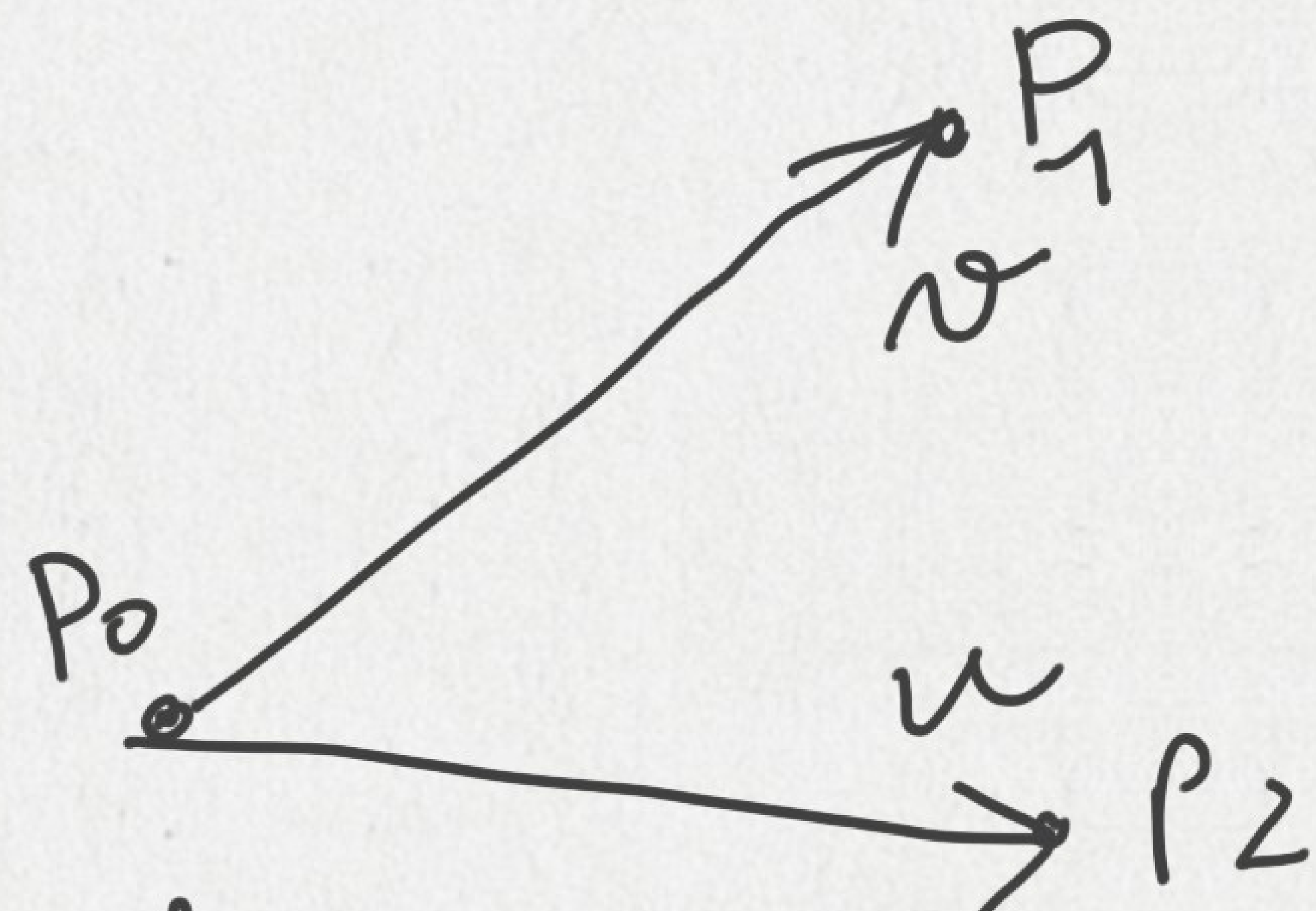
$$3(x+2) - 2(y-0) - 1(z+3) = 0$$

$$\boxed{3x - 2y - z + 9 = 0}$$

Eq cartesiana a paramétrica

$$\boxed{3x - 2y + 4z = 8}$$

$\vec{n} = (3, -2, 4)$



Eq plano

$y = 0,\ z = 0 \implies x = 8/3 \qquad P_0 = (8/3, 0, 0)$

$x = 0,\ y = 0 \implies z = 2 \qquad P_1 = (0, 0, 2)$

$x = 0,\ z = 0 \implies y = -4 \qquad P_2 = (0, -4, 0)$

$\overrightarrow{P_0P_1} = (-8/3, 0, 2) = v$

$\overrightarrow{P_0P_2} = (-8/3, -4, 0) = u$

$P = P_0 \cancel{+} tu + sv$

$$\begin{cases} x = 8/3 - 8/3\,t - 8/3\,s \\ y = 0 - 4t + 0s \\ z = 0 + 0t + 2s \end{cases}$$

Eq parametrica

do Plano

Eq paramétrica / Eq simétrica

E. cracao

- reta ⇒ dado p. dois pontos

Plano dado por 3 pontos

Eq cartesiana

Parametrica